A Probabilistic Approach to Planning and Control in Autonomous Urban Driving

Michael P. Vitus
Electrical Engineering and Computer Sciences
University of California at Berkeley
Email: michael.vitus@gmail.com

Claire J. Tomlin
Electrical Engineering and Computer Sciences
University of California at Berkeley
Email: tomlin@eecs.berkeley.edu

***Abstract*— This paper considers the problem of decision making and control for autonomous urban vehicles operating among other non-cooperating, possibly human controlled, vehicles. The difficulty in this problem stems from the fact that the behavior of the other vehicles is uncertain, and in many circumstances a collision cannot be prevented even under restrictive assumptions about the other drivers' actions. Traditional approaches that consider the worst-case actions of the other vehicles typically are inapplicable because the solutions are either too conservative or infeasible. This work proposes a new method for solving this problem using a chance constrained framework, which requires that the violation probability of all the constraints is guaranteed to be below a threshold. This formulation has the benefit of not only achieving traditional objectives such as minimization of fuel or travel time, but also hedges the nominal planned maneuver against potential crashes leading from the uncertainty in the other drivers' actions. The proposed framework is demonstrated on a passing maneuver example which exhibits a region where the passing vehicle cannot be guaranteed to prevent a collision in all circumstances.**

I. Introduction

The significant progress in autonomous urban driving over the past 30 years [1], [2] contributed to recent successes including the Defense Advanced Research Projects Agency (DARPA) urban challenge [3], the VisLab intercontinental autonomous challenge [4], and the Google autonomous car [5]. Given these successes, it is speculated that within 30 years there will be autonomous cars integrated into the driving network which may eventually replace human drivers. While these demonstrations have proven the feasibility of the technology, at this time there is a large gap between those demonstrations and what is commerically available.

Currently, there are many driver assistance systems such as lane departure warning, adaptive cruise control, collision avoidance, and blind spot detection that help humans with the task of driving and increase the overall safety of the driving network. These systems have benefited from the significant progress in sensing, low-level control, and high-level route planning, however, all these systems lack the ability to perform any tactical planning which is critical to enable the future of autonomous vehicle technology.

One area were humans currently outperform autonomy is dealing with unexpected behavior or interpreting traffic situations. In order to safely integrate autonomous vehicles into urban driving, they must be able to do the same and safely account for the inherent uncertainty in the domain, which comes from the sensor noise as well as the uncertainty in the behaviors of the other vehicles. This paper focuses on how to incorporate predictions of the possible driver actions into the planning problem to compute the tactical strategy. As stated in [6], "formally including these [human behavior] predictions into planning mostly remains an open question."

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In driving, there are many situations that arise which require the driver (human or autonomous) to plan their actions under uncertainty of the behavior of the other vehicles. Given the large action space and the close proximity of the other vehicles, one cannot use an approach that considers the worst-case actions because the solution would either be too conservative or infeasible.

California Partners for Advanced Transportation Technology [7] and Safe Road Trains for the Environment [8] have proposed the idea of platooning to automate highway driving to increase safety and fuel efficiency, however, this concept limits the potential applicability of autonomous vehicles.

Researchers have also investigated less restrictive solutions. The work in [9] proposes a machine learning approach for tactical reasoning in traffic situations such as passing maneuvers and merging into a traffic stream. A hybrid control approach to semiautonomous multivehicle safety is developed in [6]. Verma and Del Vecchio propose to model human driving behavior as a hybrid automaton with each mode representing a driving primitive such as accelerating or braking. Using an estimate of the driving mode, they develop a controller based upon reachable sets to derive the least restrictive safe control actions.

In this paper, the approach taken to handle the uncertainty in the behavior of other drivers is to plan in the belief space (the space of probability distributions of the system). One approach to planning in the belief space is to use *chance constrained programming* introduced by Charnes, Cooper and Symonds [10]. This formulation allows constraints with non-deterministic constraint parameters, while only guaranteeing constraint satisfaction up to a specified limit. In recent years, chance constrained programming has received a lot of attention [11], [12], [13].

Another related topic that has been extensively studied [14] is the feedback control problem for jump Markov linear systems, which can be used to model the uncertainty in driving behavior. This previous work was primarily concerned with solving the feedback control problem to minimize the expected value of a cost function given the value of the state and mode at the current time-step. In contrast, this paper is concerned with the predictive stochastic control problem which takes into account the future distribution of the system in order to control it to satisfy the system's constraints. This problem is difficult because it not only requires the solution to minimize traditional objectives such as fuel or travel time but also to hedge the nominal planned maneuver against potential bad actions from other drivers.

This work proposes a new framework for tactical planning in autonomous urban driving by modeling it as a chance constrained optimization problem. While this approach has the benefit of directly accounting for the uncertainty in the behavior and decisions of the other drivers, it does not readily lead to a tractable problem formulation. Given the discrete and continuous actions of the other vehicles [6], the belief space of the future system state is a multimodal distribution which makes it difficult to evaluate the probability distribution as well as enforce the chance constraints. In addition, the collision avoidance constraints result in a nonconvex feasible region resulting in a nonconvex optimization program. To overcome these difficulties, this work proposes to use sampling to represent the system state and to use either the convex bounding (conditional value-at-risk) method or hybrid method [13] to handle the chance constraints. To deal with the nonconvex feasible regions in the collision avoidance problem, each sample is considered separately to determine the correct high-level action which transforms the feasible region into a convex region. This new framework is demonstrated on an example of a passing maneuver to highlight the success of the algorithm.

II. System Model and Control Strategy

The problem considered has an autonomous vehicle operating among a set of non-cooperating vehicles which can either be human- or autonomous-controlled. These will be referred to as human-controlled without loss of generality. It has been shown that humans performing structured tasks (such as driving, drawing, etc.) can be accurately modeled as hybrid dynamical systems [6]. Given this, the human driver will be modeled using a jump Markov linear system formulated as a discrete-time stochastic hybrid system as illustrated in Figure 1(a). The system state at time-step k contains both a continuous state x_k and a discrete mode σ_k. The continuous state and output of the stochastic switched system evolve according to

$$x_{k+1}=A_{\sigma_k}x_k+B_{\sigma_k}u_k+W_{\sigma_k}w_k, \quad y_k=C_{\sigma_k}y_k+V_{\sigma_k}v_k, \quad (1)$$

where w_k and v_k are the process and output noises with known probability distributions $p(w_k)$ and $p(v_k)$. The discrete mode $\sigma_k \in \{1,\ldots,M\}$ is a Markov chain that evolves according to

$$\mathrm{P}(\sigma_{k+1}=j|\sigma_k=i)=T_{ij}, \quad (2)$$

where $T \in \mathbb{R}^{M\times M}$ is the transition matrix. The initial distribution of the discrete and continuous state is assumed to be known and given by $p(x_0,\sigma_0)$. The discrete mode is assumed to be independent of the continuous state and continuous input.

Given the independence of the discrete and continuous states, the probability distribution of the hybrid continuous-discrete state is

$$\mathrm{P}\left(\mathbb{X},\boldsymbol{\sigma}|\mathbb{U},\mathbb{Y}\right)=\mathrm{P}\left(\mathbb{X}|\boldsymbol{\sigma},\mathbb{U},\mathbb{Y}\right)\mathrm{P}\left(\boldsymbol{\sigma}\right), \quad (3)$$

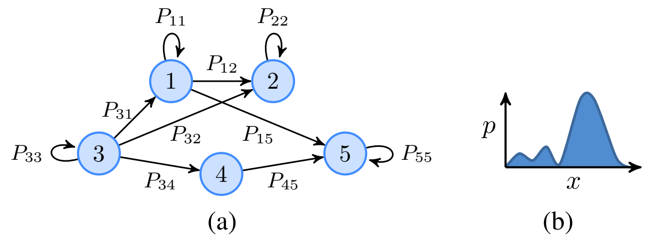


Fig. 1. (a) A graphical model of a jump Markov linear system. (b) The multimodal distribution of the future system state.

where $\boldsymbol{\sigma}=[\sigma_0,\ldots,\sigma_N]^{\mathrm{T}}$ and N is the finite time horizon. In general, even if all the continuous distributions, $\mathrm{P}\left(\mathbb{X}|\boldsymbol{\sigma},\mathbb{U},\mathbb{Y}\right)$, are Gaussian, the total distribution is a multimodal, non-Gaussian distribution due to mode switching as shown in Figure 1(b). This can be determined from the system state probability being the sum of weighted Gaussian distributions for each mode. Given this multimodal distribution, it is difficult to apply standard techniques to solve the planning in the belief space problem.

There are many approaches for making decisions under uncertainty. One approach is to assume worst-case uncertainty for the human driver and prevent a collision under any possible action of the human driver. Unfortunately, for the task of driving, the set of possible actions is too broad which causes the control strategy to be very conservative if not non-existent due to the fact that there is no strategy which can prevent a collision if the human driver tries to force one. Another approach is to limit the set of possible actions. This can be accomplished by either making assumptions on typical driving behavior or by estimating the possible actions of the driver based upon the observations of the driver. While this approach will be less conservative than assuming worst-case actions for the human driver, it may still lead to non-existent control strategies because the set of possible actions is still too broad to prevent a collision in all circumstances.

Instead of guaranteeing (up to the assumptions in the problem formulation) safety regardless of the actions of the other driver, it may be advantageous to constrain the probability of a collision occurring to be below some threshold, i.e. $\mathrm{P}\left(\text{collision}\right) \leq \delta$. This approach has two main benefits. First, by using a probabilistic measure of safety, the approach will reduce the amount of conservatism in the solution and will enable previously infeasible solutions if they result in a low chance of leading to a collision. Second, this framework also provides a way to systematically tradeoff between the performance and safety of the system. In addition, it specifically takes into account potential bad actions of other drivers and hedges the nominal planned maneuver to help mitigate them.

While using a probabilistic measure of safety has its benefits, it still raises a few concerns. The first concern involves the accuracy of the probability distributions over the actions of the other drivers. It may be argued that if they are wrong, then this approach provides no benefit. However, a similar argument can be made about the provably safe methods; if their assumptions are wrong then their guarantees may not hold. The concern over the accuracy of the model is a very common problem in control; in this discussion, it is appropriate to consider the following famous quote:

All models are wrong, but some are useful.

George Box

It is almost certainly true that the probability distributions over the actions of the other drivers are not 100% correct, however, the emergent behavior from planning in the belief space will still exhibit useful behavior. For example, the vehicle will not choose actions that lead to serious outcomes. When approaching a car on the right in a two lane highway with its left indicator on, it will slow down until it estimates the indicator was inadvertently turned on. In addition, it can consider worst-case actions of the other drivers without resulting in an infeasible problem, this allows the ability to hedge against these actions to reduce their affect if they were to occur. Finally, by using a probabilistic measure of safety it allows the ability to make high level decisions on whether or not to perform a specific maneuver based upon the risk.

Another issue is the practical problem of determining the prior probabilities over the actions of human drivers. While many years ago this would be a daunting (if not infeasible) task, the crowd sourced driving data being collected via smartphones [15] can be used to analyze the driving situations to build up the required probability distributions.

One way of using a probabilistic measure of safety is through chance constrained control. This will be formally developed in the following section, with the application to a car passing scenario.

III. Chance Constrained Control Solution

The chance constrained stochastic control problem can be expressed as

$$\begin{array}{ll} \text{minimize} & \mathbf{E}\left[\phi(\mathbb{X},\mathbb{U})\right] \\ \text{subject to} & \\ & \mathbb{X} = f(x_0, \boldsymbol{\sigma}, \mathbb{U}, \mathbb{W}) \\ & \mathbb{Y} = h(\mathbb{X}, \boldsymbol{\sigma}, \mathbb{V}) \\ & \mathrm{P}\left(\varphi\left(\mathbb{X},\mathbb{U}\right) \le 0\right) \ge 1-\delta \end{array} \tag{4}$$

where the expectation is over the noise sources $\mathbb{W}$ and $\mathbb{V}$ and the optimization is over the control policy. In general, this optimization program is nonconvex. For notational convenience, the system state ($\mathbb{X}$), measurements ($\mathbb{Y}$), control inputs ($\mathbb{U}$), process noise ($\mathbb{W}$) and measurement noise ($\mathbb{V}$) for all time-steps are concatenated to form a column vector. Also, $\mathbb{X} = f(x_0, \boldsymbol{\sigma}, \mathbb{U}, \mathbb{W})$ and $\mathbb{Y} = h(\mathbb{X}, \boldsymbol{\sigma}, \mathbb{V})$ will be used as a compact representation for the calculation of all states and measurements.

There is also a set of constraint functions on the state trajectory and control inputs, $\varphi : \mathbb{R}^{nN} \times \mathbb{R}^{mN} \to \mathbb{R}^q$, which are assumed to be convex functions. Unfortunately, due to the stochasticity of the problem, the system constraints cannot be considered deterministically; the stochasticity may result in a non-zero chance that the constraints will be violated. Consequently, the constraints must be considered probabilistically, leading to a notion of risk. In this work, risk is modeled as the probability of violating the constraints leading to the use of chance constraints.

The main challenges to solving the optimization program (4) is in characterizing the distribution of the system state and control inputs as well as satisfying the chance constraints on the state and control inputs. The following subsection introduces one approach of dealing with the chance constraints to form a convex optimization program, and the remaining sections demonstrate how to formulate solutions to specific problems for decision making under uncertainty for autonomous driving.

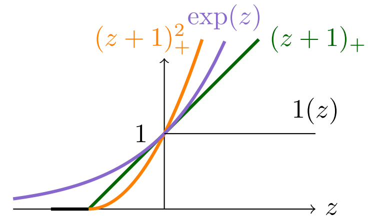


Fig. 2. A comparison of several convex bounds that can be used to approximate the indicator function $\mathbf{1}(z)$.

A. Convex Bounding Method

One approach to handling the complexity of the stochastic hybrid system used to model the driver is by using the convex bounding method developed by [16], [17]. Since the probability distribution of the chance constraints is a multimodal function, it is difficult to include them in the optimization program. The intuition behind this method is to find a conservative, convex approximation for the chance constraint to convert the problem into a convex program.

To illustrate this method, consider a single individual chance constraint of the form $\mathrm{P}\left(\varphi_i\left(\mathbb{X},\mathbb{U}\right) > 0\right) \le \delta_i$, which can be calculated via

$$\mathrm{P}\left(\varphi_i\left(\mathbb{X},\mathbb{U}\right) > 0\right) = \mathrm{E}\left[\mathbf{1}\left(\varphi_i\left(\mathbb{X},\mathbb{U}\right)\right)\right] \tag{5}$$

where $\mathbf{1}(\cdot)$ is the indicator function defined as

$$\mathbf{1}(z) = \begin{cases} 1, & \text{if } z > 0 \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

From this representation, it can be easily seen that the chance constraint in problem (4) is nonconvex since the indicator function $\mathbf{1}(z)$ is a nonconvex function; this greatly complicates solving the optimization program. To reduce the complexity, this method bounds the indicator function with a convex function thereby rendering the program convex.

Suppose a nonnegative, nondecreasing, convex function $\psi : \mathbb{R} \to \mathbb{R}$ can be found such that for any $\alpha > 0$, $\psi(z/\alpha) \ge \mathbf{1}(z)$ for all z then

$$\mathrm{E}\left[\psi\left(\varphi_i\left(\mathbb{X},\mathbb{U}\right)/\alpha\right)\right] \ge \mathrm{P}\left(\varphi_i\left(\mathbb{X},\mathbb{U}\right) > 0\right). \tag{7}$$

Consequently, the following convex constraint can be used in place of the original chance constraint in Eqn. (6):

$$\mathrm{E}\left[\psi\left(\varphi_i\left(\mathbb{X},\mathbb{U}\right)/\alpha\right)\right] \le \delta_i, \tag{8}$$

which will guarantee the original constraint holds.

The next step is to determine the exact form of function to use for ψ. In this work, $\psi(z)$ was chosen to be $\psi(z) = (z+1)_+$ (where the subscript $+$ denotes $\max\{z+1, 0\}$) because it results in the least conservative bound. Using this function, the final convex constraint is:

$$\mathrm{E}\left[\left(\max\left(\varphi\left(\mathbb{X},\mathbb{U}\right)\right) + \alpha\right)_+\right] \le \alpha\delta, \tag{9}$$

and the $\max$ is introduced to handle joint chance constraints.

By using the indicator function in Eqn. (6), the original chance constraints equally weight all violations of the constraints no matter how large. In contrast, the convex bounding methods use functions that approximate the indicator function and weight larger constraint violations more than smaller ones. This can be visualized in Figure 2. For the application of car driving, this non-equal weighting is beneficial because the amount of constraint violation is likely proportional to the damage that would occur in a collision.

Unfortunately, given the choice of bounding function $\psi(z)$, there is no analytical expression to compute the expectation in Eqn. (9). To overcome this, it can be approximated through sampling. One advantage of using sampling is that it can represent the multimodal distribution of the human driver's behavior.

The expectation in Eqn. (9) can be evaluated by drawing N_s particles from the process noise, measurement noise, initial state, and discrete mode at each time-step to obtain: $\{w_0^{(1)}, \ldots, w_{N-1}^{(1)}, \ldots, w_0^{(N_s)}, \ldots, w_{N-1}^{(N_s)}\}$, $\{v_0^{(1)}, \ldots, v_{N-1}^{(1)}, \ldots, v_0^{(N_s)}, \ldots, v_{N-1}^{(N_s)}\}$, $\{x_0^{(1)}, \ldots, x_0^{(N_s)}\}$, $\{\boldsymbol{\sigma}^{(1)}, \ldots, \boldsymbol{\sigma}^{(N_s)}\}$. The expectation is calculated over both the system state and the discrete mode as follows,

$$\begin{aligned} &\mathrm{E}\left[(\max\left(\varphi\left(\mathbb{X}, \mathbb{U}\right)\right) + \alpha)_+\right] \approx \\ &\frac{1}{N_s} \textstyle\sum_{j=1}^{N_s} \left(\max\left(\varphi\left(\mathbb{X}^{(j)}\left(\boldsymbol{\sigma}^{(j)}\right), \mathbb{U}^{(j)}\right)\right) + \alpha\right)_+, \end{aligned} \tag{10}$$

where the state dependence on the discrete mode is explicitly expressed. Using the convex bounding method, the final optimization program is

$$\begin{aligned} \text{minimize} \quad & \mathbf{E}\left[\phi(\mathbb{X}, \mathbb{U})\right] \\ \text{subject to} \quad & \\ & \mathbb{X}^{(i)} = f(x_0^{(i)}, \boldsymbol{\sigma}^{(i)}, \mathbb{U}^{(i)}, \mathbb{W}^{(i)}), \quad \forall i \\ & \mathbb{Y}^{(i)} = h(\mathbb{X}^{(i)}, \boldsymbol{\sigma}^{(i)}, \mathbb{V}^{(i)}), \quad \forall i \\ & \mathrm{E}\left[(\max\left(\varphi\left(\mathbb{X}, \mathbb{U}\right)\right) + \alpha)_+\right] \le \delta \end{aligned} \tag{11}$$

IV. Case Study: Passing a Tractor Trailer

In this example, the proposed framework is used to analyze the scenario of passing a tractor trailer on a four-lane highway. This maneuver is interesting because during the maneuver there is a region in which the passing vehicle cannot prevent a collision.

A. Formulation

The system dynamics for the passing car and tractor trailer are modeled in relative coordinates using discretized point mass dynamics,

$$\begin{aligned} x_k &= \begin{bmatrix} \Delta p_k \\ \Delta v_k \\ l_k \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \Delta t \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-K \end{bmatrix}, \qquad B_{1,2} = \begin{bmatrix} 0.5\Delta t^2 \\ \Delta t \\ 0 \end{bmatrix}. \end{aligned} \tag{12}$$

Let Δp_k be the relative longitudinal position between the two vehicles with $\Delta p_k \le 0$ and $\Delta p_k \ge L$ indicate that the passing car is behind and ahead of the tractor trailer, respectively, let Δv_k be the relative longitudinal velocity, let l_k be the lateral position of the tractor trailer, and let K be a gain that dictates the lane changing behavior. Define Δl_k to be the difference between the lateral positions of the tractor trailer and passing car with $\Delta l_k \ge W$ indicating that the lateral positions do not overlap. It is assumed that there are a set of constraints on the minimum $(\underline{\Delta v_\mathrm{b}}, \underline{\Delta v_\mathrm{a}})$ and maximum $(\overline{\Delta v_\mathrm{b}}, \overline{\Delta v_\mathrm{a}})$ relative velocities where the subscript 'b' and 'a' represent before and after the tractor trailer decides to change lanes. The different set of bounds on the velocities before and after is to model the behavior that drivers exhibit a wider range of feasible actions when trying to avoid a conflict. A constraint on the feasible inputs is also imposed $\underline{u} \le u_k \le \overline{u}, \forall k$.

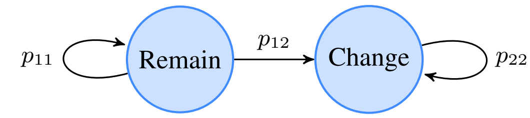


Fig. 3. The jump Markov model for the tractor trailer example. There are two nodes: remaining in the current lane, or performing a lane change. If the tractor trailer changes lanes then it is assumed it completes the maneuver.

This example is modeled by a hybrid system, as shown in Figure 3, with two states: 1) the tractor trailer remains in the current lane, 2) the tractor trailer changes lanes. In this model, there is a probability $1-p$ that the tractor trailer will initiate a lane change and once a lane change is initiated it must be completed. An abort lane change mode could also be included in the model, but it was omitted to obtain a more robust solution. In this problem, a collision doesn't occur if

$$\Delta l_k > W \vee \Delta p_k \le 0 \vee \Delta p_k \ge L, \forall k. \tag{13}$$

The passing maneuver can be examined to determine the set of possible actions that the passing vehicle has which can prevent a collision with the tractor trailer. To perform this analysis, the backwards reachable set was calculated from the set of final safe conditions of behind or ahead of the tractor trailer. Figure 4 illustrates the set of safe and unsafe initial conditions, where the x-axis is the initial relative speed and the y-axis is the relative distance between the two vehicles when the tractor trailer initiates the lane change. From the analysis, if the tractor trailer initiates the lane change while the passing car is in the red region, then there is nothing the passing vehicle can do to prevent a collision. In the dark and light blue regions, the passing vehicle can either accelerate or decelerate, respectively, in order to prevent a collision. From this analysis, there is no trajectory through the state space that can guarantee the safety of the passing car. Furthermore, due to the tractor trailer's blindspots, the trailer could inadvertently change lanes into the vehicle, leading to an unavoidable imminent collision. Consequently, if an algorithm that guaranteed the safety of the system for any action of the tractor trailer was used, it would never allow the vehicle to pass the tractor trailer causing the system to be in a deadlock condition. Therefore, the only way to prevent this deadlock condition is for the passing car to allow some amount of risk of a collision. Given the reachable set, it is not apparent what the best trajectory in $(\Delta v, \Delta p)$ space is to minimize the probability of collision. A novel method of simultaneously making the high-level decision under the uncertainty of the tractor trailer as well as calculating the continuous control inputs to hedge against the possible actions of the tractor trailer is further formulated below.

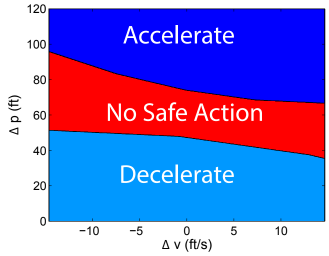


Fig. 4. The backwards reachable set for the tractor trailer passing maneuver. The x-axis is the initial relative speed and the y-axis is the relative distance between the two vehicles when the tractor trailer initiates the lane change.

The problem is formulated as the program (14).

$$\begin{array}{ll} \text{minimize} & \mathbf{E}\left[\phi(\mathbb{X},\mathbb{U})\right] \\ \text{subject to} & \\ & x_{k+1} = A_{\sigma_k} x_k + B u_k \\ & \underline{u} \le u_k \le \overline{u} \\ & \underline{\Delta v_{\mathrm{b}}} \le \Delta v_k \le \overline{\Delta v_{\mathrm{b}}}, \text{ if } \sigma_k = 1 \\ & \underline{\Delta v_{\mathrm{a}}} \le \Delta v_k \le \overline{\Delta v_{\mathrm{a}}}, \text{ if } \sigma_k = 2 \\ & \mathrm{P}(\Delta l_k \ge W \vee \Delta p_k \le 0 \vee \Delta p_k \ge L\ \forall k) \ge 1-\delta \end{array} \tag{14}$$

The optimization program is nonconvex for two reasons. First, the stochastic nature of the discrete state results in a multimodal distribution of the future system state which causes any chance constraint to be nonconvex. This can be mitigated by using sampling to represent the probability distribution. For this example, a set of samples is drawn from the distribution of the discrete mode of the hybrid system which represents the uncertainty in the tractor trailer's decision of changing lanes: $\{\boldsymbol{\sigma}^{(1)},\ldots,\boldsymbol{\sigma}^{(N_s)}\}$. It is assumed that there is not any significant noise in the system state, but it could also be included in the problem solution. This sampling technique eliminates the first problem of not being able to represent the probability distribution through analytical functions.

The second reason for nonconvexity is the feasible region for the passing vehicle is nonconvex. The chance constraint $\mathrm{P}(\Delta l_k \ge W \vee \Delta p_k \le 0 \vee \Delta p_k \ge L\ \forall k) \ge 1-\delta$ results in a disjunction of chance constraints:

$$\begin{array}{l} \mathrm{P}(\Delta l_k \ge W) \ge 1-\delta_k \vee \mathrm{P}(\Delta p_k \le 0) \ge 1-\delta_k \vee \\ \mathrm{P}(\Delta p_k \ge L) \ge 1-\delta_k, \sum \delta_k \le \delta, \end{array} \tag{15}$$

which is a nonconvex constraint. To solve this problem, the chance constraint might be simplified by utilizing information from the set of samples. Each sample, $\boldsymbol{\sigma}^{(i)}$, corresponds to a time k_{lc} (which can be infinity) that the tractor trailer will change lanes. From the reachable set in Figure 4, there is a distinct separation of when the passing car should either abort or continue the maneuver. Consequently, if this can be converted into a time-step k_{dp} that represents this decision point then it can be used to simplify the chance constraint. In this scenario, the time-step decision point k_{dp} can be calculated using the maximum deceleration/acceleration rates and comparing the constraint violation at various time-steps. Let N_{dur} represent the number of time-steps it takes the tractor trailer to collide with the passing car if an accident cannot be prevented. Using this information, the chance constraint in Eqn. (15) can be simplified to

$$\begin{array}{ll} \mathrm{P}(1) \ge 1-\delta_k, & \text{if } k_{lc} = \infty \\ \mathrm{P}(\Delta p_{k_c+k} \le 0) \ge 1-\delta_k, & \text{if } k_{lc} \le k_{dp} \\ \mathrm{P}(\Delta p_{k_c+k} \ge L) \ge 1-\delta_k, & \text{otherwise,} \end{array} \tag{16}$$

where $k_c = k_{lc} + N_{dur}$ and $\sum_{k=0}^{N-k_c} \delta_k \le \delta$. Now that the constraints have been decomposed into convex constraints, the chance constraint can now be formulated.

Combining both the sampling and the simplification of the feasible region of the passing car, the convex bounding method can be used to enforce the chance constraint. Let $c^{(i)}$ be the constraint for each sample defined as

$$c^{(i)} = \begin{cases} -\infty, & \text{if } k_{lc} = \infty \\ \max\limits_{k \in \{k_c,\ldots,N\}} \Delta p_k, & \text{if } k_{lc} \le k_{dp} \\ \max\limits_{k \in \{k_c,\ldots,N\}} L - \Delta p_k, & \text{otherwise.} \end{cases} \tag{17}$$

Finally, using the convex bounding method the chance constraint is

$$\frac{1}{N_s}\mathbf{1}^{\mathrm{T}}\left([c^{(1)},\ldots,c^{(N_s)}]^{\mathrm{T}} + \alpha\mathbf{1}\right)_+ \le \alpha\delta, \tag{18}$$

where $\mathbf{1}$ is a column vector of all ones with size N_s.

The final optimization program is shown in Eqn. (19).

$$\begin{array}{ll} \text{minimize} & \mathbf{E}\left[\phi(\mathbb{X},\mathbb{U})\right] \\ \text{subject to} & x_{k+1}^{(i)} = A_{\sigma_k^{(i)}}^{(i)} x_k^{(i)} + B u_k^{(i)} \\ & \underline{u} \le u_k^{(i)} \le \overline{u} \\ & \underline{\Delta v_{\mathrm{b}}} \le \Delta v_k^{(i)} \le \overline{\Delta v_{\mathrm{b}}}, \text{ if } \sigma_k^{(i)} = 1 \\ & \underline{\Delta v_{\mathrm{a}}} \le \Delta v_k^{(i)} \le \overline{\Delta v_{\mathrm{a}}}, \text{ if } \sigma_k^{(i)} = 2 \\ & \frac{1}{N_s}\mathbf{1}^{\mathrm{T}}\left([c^{(i)},\ldots,c^{(N_s)}]^{\mathrm{T}} + \alpha\mathbf{1}\right)_+ \le \alpha\delta \end{array} \tag{19}$$

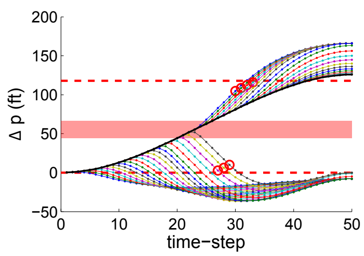
This optimization could be infeasible for a given probability of failure δ and hybrid mode probability estimate p, however, this means that the passing maneuver is currently not safe. Consequently, the vehicle needs to maintain its current safe relative position until the estimator of the human behavior changes enough to allow the passing maneuver.

B. Results

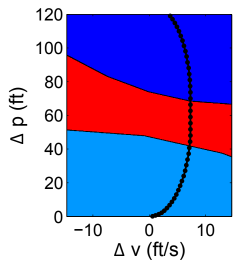
The problem was solved for $N_s = 1045$ samples with an allowed probability of failure of $\delta = 0.015$, and the results are shown in Figure 5. For this example, the transition probability is $p = 0.999$, $\Delta t = 0.5$ seconds, and the following limits are imposed $\underline{\Delta v_{\mathrm{b}}} = -5$ MPH, $\overline{\Delta v_{\mathrm{b}}} = 5$ MPH, $\underline{\Delta v_{\mathrm{a}}} = -10$ MPH, $\overline{\Delta v_{\mathrm{a}}} = 10$ MPH, $\underline{u} = -27.65\text{ft/s}^2$, $\overline{u} = 14.8$ ft/s^2. The acceleration and deceleration limits were calculated from common vehicle performance characteristics. For this experiment, the time-step decision point of whether to abort or continue the maneuver was chosen to be $k_{dp} = 22$. The cost function is a weighted function of the L_2-norm of the nominal control input and the distance from the initial position for samples that aborted the passing maneuver. This was chosen to take into account the possibility of colliding with vehicles behind the passing car. Note, this is only one choice of objective function, and the formulation is not limited to this choice of objective function.

The relative position of the vehicles versus time-step is shown in Figure 5(a). The colored lines represent the trajectories of the samples that have a lane change and the black line is the trajectory of the system with no lane change. The red circles indicate the relative positions of the vehicles when a collision would occur for the samples that lead to a collision. The red patch illustrates the region that the passing vehicle cannot prevent a collision if the tractor trailer initiates a lane change. For the 1045 samples only 41 have a lane change and 7 of them lead to a collision between the vehicles. For this solution, the true probability of collision is 0.0067 which is below the allowed 0.015. One important result of this solution method is that it simultaneously plans the overall passing maneuver behavior while attempting to mitigate the consequences of potential accidents.

Figure 5(b) shows the system trajectory with no lane change overlaid on the reachable set. It is interesting to note that the system doesn't choose the shortest trajectory through the unsafe region, but rather takes a curved path.



(a)



(b)

Fig. 5. (a) The relative position versus time-step for the samples with a lane change are the colored lines. The trajectory of the system with no lane change is the black line. The red dotted line represents the length of the tractor trailer. The red circles indicate the relative positions of the vehicles when a collision would occur for the samples that lead to a crash. The shaded red patch indicates the region which the passing car cannot prevent a collision. (b) The trajectory of the system (black line) with no tractor trailer lane change is overlayed onto the reachable set.

In the previous example, the time for the tractor trailer to change lanes (N_{dur}) was considered a known constant, but this formulation can also handle uncertainty in that parameter at the expense of requiring more samples to accurately represent the underlying probability distribution. Figure 6 shows the results for $N_s = 5096$ samples and a uniform distribution over $N_{dur} \in \{5, 6, 7, 8, 9\}$. The passing maneuver is similar since the distribution for the lane change maneuver is symmetric around the previous example, but the avoidance maneuver is different since the time to collision is different. In this example, there are 195 lane change maneuvers and of them 45 lead to a crash resulting in a true probability of failure of 0.0088.

V. Conclusion

The tactical planning problem for autonomous urban vehicles was formulated as a chance constrained optimization problem. This formulation has the benefit of not only minimizing traditional objectives such as fuel or travel time, but also specifically taking into account potential bad actions of other drivers and hedging the nominal planned maneuver against them. This framework was demonstrated on a vehicle passing maneuver which exhibits a region where the passing vehicle cannot prevent a collision in all circumstances.

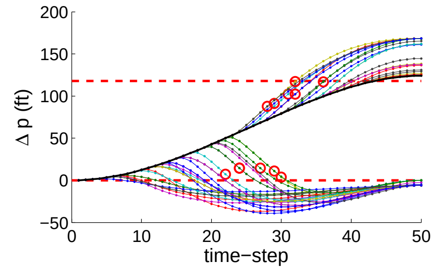


Fig. 6. The relative position versus time-step for a subset of samples with lane change (colored) and no lane change (black) for the system with variable lane change speed.

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